

SENIOR MATHEMATICS COMPETITION 2009

Preliminary round
Friday 22nd May 2009
Time allowed 1 ½ hours

Instructions

Attempt all questions. It is not expected that you will finish them all.

Full working should accompany all solutions.

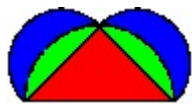
Calculators may be used, but no other reference material is permitted.

Diagrams are not necessarily drawn to scale; they are presented as aids only.

Total: 50 marks

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YOUR MATHEMATICAL SOLUTION

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1. What is the units digit, ie the last digit, in

$$(1!)^3 + (2!)^3 + (3!)^3 + \dots + (2008!)^3 + (2009!)^3 \quad [3]$$

Note: $4! = 4 \times 3 \times 2 \times 1$, $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ etc

2. How many different values can the angle ABC take, where A, B and C are distinct vertices of a cube. [3]

3. Evaluate
$$\sqrt[3]{\frac{1 \times 2 \times 4 + 2 \times 4 \times 8 + 3 \times 6 \times 12 + \dots}{1 \times 3 \times 9 + 2 \times 6 \times 18 + 3 \times 9 \times 27 + \dots}}$$
 [3]

4. Solve for a and b in $\log 96 = a \log 24 + b \log 36$ [4]

5. If the number of sides of a regular polygon is multiplied by five, the sum of the interior angles of the new polygon is a multiple of the sum of the interior angles of the original polygon.

For which regular polygon does this occur ?

[5]

6. Function $f(x)$ obeys the rule $f(x+1) = f(x) - f(x-1)$.

Given $f(2) = 5$ and $f(1) = 2$, find $f(2009)$.

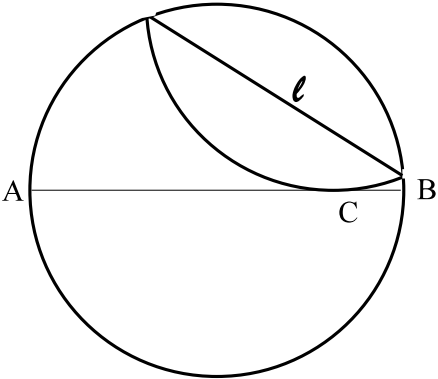
[6]

7. A bin contains 20 balls: 8 red, 7 black and 5 blue .
 We draw three balls at random (without replacement) from the bin and we say we “win” if our three balls have only two colours. (i.e we “win” if we draw two balls of one colour and another ball of a different colour.)

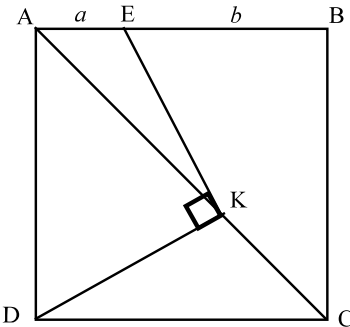
What is the probability of winning this particular game? [5]

8. Two real numbers, x and y , satisfy the condition $x + y = 2$.

Show $xy(x^2 + y^2) \leq 2$. [5]

9.  A segment of a circle is folded so that its arc is tangent to the diameter AB, at point C .
 The diameter is divided so that the ratio $AC : CB = 3:1$.
 Calculate the length of the chord , ℓ .

[6]

10.  ABCD is a square with E a point on AB such that $AE : EB = a:b$.
 K is a point such that angle $DKE = 90^\circ$.

Find the ratio $AK : KC$ in terms of a and b .

[10]