

# SENIOR MATHEMATICS COMPETITION 2010

Preliminary round  
Thursday 20<sup>th</sup> May 2010  
Time allowed 1 ½ hours

*Instructions*

*Attempt all questions. It is not expected that you will finish them all.*

*Full working should accompany all solutions.*

*Calculators may be used, but no other reference material is permitted.*

*Diagrams are not necessarily drawn to scale; they are presented as aids only.*

*Total: 52 marks*

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# SENIOR MATHEMATICS COMPETITION 2010

1. Find the value of  $n$  given that

$$(10^{2010} + 25)^2 - (10^{2010} - 25)^2 = 10^n \quad [3]$$

2. How many six-digit numbers are there in which the sum of the squares of the digits is 12?  
(Note: the six-digit number cannot start with zero.) [7]

3. There are 2010 dots evenly distributed around a circle.

If 4 different dots are chosen at random and labelled  $A, B, C$  and  $D$  as they were chosen, what is the probability that the segments  $AB$  and  $CD$  will intersect?

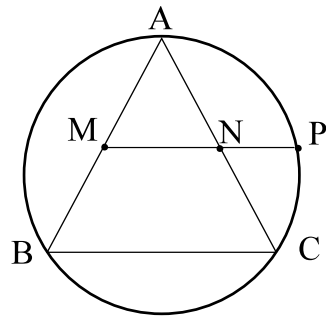
[3]

4. An equilateral triangle is inscribed in a circle.

Points  $M$  and  $N$  are the mid-points of  $AB$  and  $AC$  respectively.

Line segment  $MN$  is extended to meet the circumference at  $P$ .

Find the ratio of  $MN:NP$ .

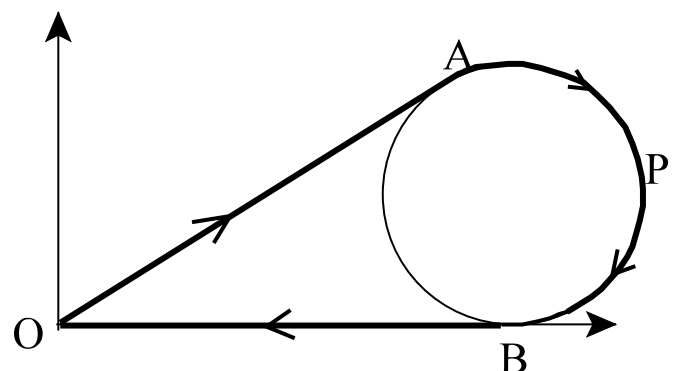


[4]

5. Steve is planning a cross-country run for his club.

He plans a course, starting at  $O$ , that follows the arrows from  $O$  to  $A$ , around the arc  $APB$  which is part of a circle which can be represented by the equation  $(x - 12)^2 + (y - 5)^2 = 25$  then from  $B$  back to the starting point  $O$ .  $OA$  and  $OB$  are tangents to the circle. What is the total length of the run.

(All distances are in km)



[5]

6. If  $\log_8 a + \log_4 b^2 = 5$  and  $\log_8 b + \log_4 a^2 = 7$ , find  $a$  and  $b$ . [6]

7. A function,  $f$ , is defined as  $f(n) = an + b$  where  $a$  and  $b$  are integers.

$f(3n + 1)$ ,  $f(3n) + 1$  and  $3f(n) + 1$  are three consecutive integers in some order.

Determine all possible functions  $f(n)$ . [5]

8. Solve for  $x$  where  $x$  is a real number.

$$(x^2 - 3x + 1)^{x+1} = 1$$
 [4]

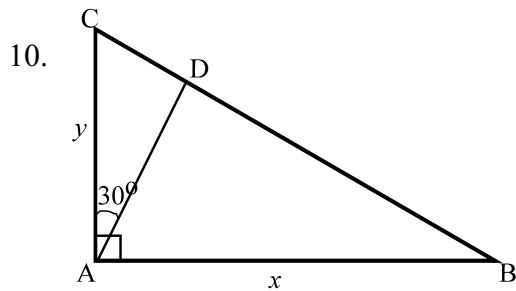
9.  $a$ ,  $a + d$ ,  $a + 2d$ ,  $a + 3d$  and  $a + 4d$  are the first five terms of an arithmetic sequence.

Given that :  $P$  is the product of the odd number terms of the sequence.

$Q$  is the product of the even numbered terms of the sequence.

$\frac{P}{Q}$  is the sum of the five terms,  
and the middle term is 5 times a perfect square.

Prove that the product of all five terms is also a square number. [5]



ABC is a right angled triangle with  $\angle BAC = 90^\circ$ ,  $AC = y$ , and  $AB = x$ .

D is a point on BC such that  $\angle CAD = 30^\circ$ .

Show that the length  $AD = \frac{2xy}{\sqrt{3}x + y}$

[5]

11. A rectangular sheet of paper is folded along a diagonal.

The original 4 corners of the sheet now form a trapezium with 3 sides of equal length.

How long was the shorter side of the original rectangular sheet if the longer side was 12 cm?

[5]